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HEAT TRANSFER WITH A MOVING BOUNDARY–APPLICATION TO FLUIDIZED-BED COATING

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Abstract—This study presents a theoretical relationship between coating thickness on an object immersed in a fluidized bed of coating material and the physical properties of the system.

The theory of fluidized bed coating was developed using the heat-balance integral in solving the problem of heat-conduction with a moving boundary as applied for the coating film. In developing the theory, the temperature profile within the coating film was represented by a second-degree polynomial.

The theoretical solution was compared with experimental data given in the literature. The agreement is good. On the average, the theory predicts coating thicknesses that are higher than the experimental ones by about 10 per cent. This deviation is attributed mainly to the assumption of constant object and coating film temperature made in the theory.

The heat-transfer coefficient is a major factor in the fluidized bed coating process. Coating data reported in the literature do not generally give the heat-transfer coefficient in the fluidized bed. A graphical method for estimating the heat-transfer coefficient was developed for experiments where final thickness data are reported.

NOMENCLATURE

A, $\frac{c}{k}(T_f - T_w)$, [h°F/ft²];
Bi, biot number, $\frac{h}{K} X_f$, dimensionless;
c, specific heat of coating material, [Btu/lb°F];
h, heat-transfer coefficient, [Btu/hft² F];
k, thermal conductivity, [Btu/hft° F];
t, immersion time [h];
T_f, softening point of coating material [°F];
T_w, object temperature [°F];
T, fluidized bed temperature [°F];
X_f, final coating thickness [ft];
$X(t)$, coating thickness [ft];
ρ, density of coating material [lb/ft];
α, thermal diffusivity, [ft²/h];
θ, dimensionless temperature, $\frac{T_w - T_f}{T_f - T_\infty}$.

INTRODUCTION

THE fluidized-bed system for coating metals with plastics has developed from a laboratory curiosity barely thirteen years ago to a routine process in operation today in more than 360 major companies [7]. Applications are increasing in the appliance, chemical processing, electrical, power distribution and pipeline fields.

In the fluidized-bed coating process, a fusible polymeric resin in powder form is applied to the surface of an object that is immersed in a bed or chamber of powder through which a current of gas is passed. The gas serves to levitate the resin powder in such a manner that it resembles a boiling liquid in appearance. The object is heated to a temperature high enough above the melting or softening point of the resin so that, after the object is removed from the heat source, it retains enough heat on its surface to melt the resin powder particles, which then stick fast, melt, and flow together to form a coating.

In fluidized-bed coating, there are a number of variables that can affect the thickness and uniformity of coating layers applied to the objects. The major variables affecting the thickness of coating layer are object temperature, immersion time, bed temperature, velocity of fluidizing gas, particle size, shape and size distribution of the resin powder, and the physical properties of object, powder and carrier gas.

In spite of the widespread use of the fluidized-bed coating process and the many experiments done, no coating theory has been developed to date. In the present paper, an attempt has been made to present a theory that will correlate the coating thickness with the other coating parameters.

FLUIDIZED-BED COATING—STATEMENT OF THE PROBLEM AND ASSUMPTION

The discussion presented in this paper deals with the growth of coating films on vertical plates in a fluidized bed. We consider one-dimensional heat conduction in a coating film that extends from $x = 0$ to $x = X(t)$. The face $x = 0$ is the object surface. If the surface temperature, T_i, is at or above the melting or softening point, T_f, the coating commences. If the surface temperature, T_i, of the film drops below T_f, the growth of the coating film stops and $X(t)$ remains constant. The thickness of the coating film $X(t)$ as a function of time is the quantity we wish to find.

The equation describing the process is as follows:

$$\rho c \frac{\partial T}{\partial t} = \frac{\partial}{\partial x}\left[k \frac{\partial T}{\partial x}\right] \tag{1}$$

with boundary conditions

$$T(0, 0) = T_w > T_f \tag{2}$$

$$T(0, t) = T_w(t) \tag{3}$$

$$T[X(t), t) = T_i(t) \tag{4}$$

$$-k \frac{\partial T}{\partial x}\bigg|_{x=X(t)} = h(T_i - T_\infty) + \rho c(T_i - T_\infty)\frac{\partial X(t)}{\partial t}. \tag{5}$$

Equation (5) expresses the fact that heat conduction at the surface of the coating film equals the heat flow into the fluidized bed by convection plus the heat absorbed by the bed particles that adhere to the object and form the film. In this energy balance, we neglect the heat loss by radiation. It is apparent that the convective transfer of heat far outweighs all other types of heat transfer in a fluidized bed system.

The essential difficulty in the problem is in the determination of the unknown moving boundary, $X(t)$. This is a non-linear problem because it involves a moving boundary whose location is unknown *a priori*. We were unable to treat it in an exact analytical manner; thus, we had to choose between alternative methods of either using a high-speed computer or of finding an approximate solution under some simplifying assumptions. The latter course was taken.

In the present paper the discussion is limited to cases where the following assumptions apply:

1. The thermal properties of material, ρ, c, k, are constant for a particular material during the coating.
2. The temperature within the fluidized bed is uniform throughout and constant.
3. The temperature of the particles and the fluid is the same.
4. The object temperature, T_w, is constant during the coating process.
5. The surface temperature of the coating film, T_i, is constant and equals the melting or softening point of the material, T_f.
6. The thickness of films does not depend on orientation of the coated object in the fluidized bed.
7. The heat-transfer coefficient between the object and the fluidized bed is constant during coating.
8. Changes in the heat-transfer coefficient

over the height of the object are negligibly small.

9. The existence and uniqueness of $T(x,t)$ and $X(t)$ are assumed.

Assumption 9 is adopted from the classical moving boundary problem known as the Stefan problem [3].

The most questionable assumption is No. 4. In reality the temperature of the coated object decreases during the coating process. The assumption of constant T_w is equivalent to that of a large heat reservoir kept isothermally. In many applications of fluidized-bed coating the condition of a constant-temperature reservoir can be approximated. In most cases a plastic powder which is a good insulator is used to coat a metal object—a good conductor. Thus, the assumption of constant temperature within the metal solid is reasonable. In those cases where this assumption does not hold, such as in coating of thin wires, the present solution can be viewed as an upper bound on the coating thickness.

Using the assumptions made above, the equations describing the process are:

$$\frac{\partial T}{\partial t} = \alpha \frac{\partial^2 T}{\partial x^2} \tag{6}$$

with the boundary conditions

$$T(0,0) = T_w > T_f \tag{7}$$

$$T(0,t) = T_w \tag{8}$$

$$T(X(t),t) = T_f \tag{9}$$

$$-k \left.\frac{\partial T}{\partial x}\right|_{x=X(t)} = h(T_f - T_\infty) + \rho c(T_f - T_\infty)\frac{\mathrm{d}X(t)}{\mathrm{d}t}. \tag{10}$$

These equations can be further limited to a narrower class of heat-transfer problems by neglecting the convective term in equation (10). This will be done later on in the solution of a special case, in order to demonstrate a limiting behavior of the general solution.

APPROXIMATE SOLUTION OF THE FLUIDIZED BED COATING PROBLEM

As mentioned above, the heat transfer problems involving a moving boundary are nonlinear, and, except in very special cases [1], can be solved either by using high-speed computers or by some approximate technique. In this paper, we solve the heat-transfer problem in fluidized-bed coating by using the heat balance integral method [2]. For the one-dimensional case, the equation determining the thickness of the coating film reduces to an ordinary differential equation when this method is applied. Thus, it can be solved analytically or numerically. These solutions, although not exact, are accurate enough to be of practical use. We may also note that our main interest is the determination of film thickness as a function of time, rather than the temperature distribution in the film. Minor variations in the temperature profile inside the film are of secondary importance to the build up of film thickness taking place on its surface.

Returning to the mathematical problem, we now multiply both sides of equation (6) by $\mathrm{d}x$ and integrate from $x = 0$ to $x = X(t)$.

$$\int_0^{X(t)} \frac{\partial T}{\partial t}\,\mathrm{d}x = \alpha \int_0^{X(t)} \frac{\partial^2 T}{\partial x^2}\,\mathrm{d}x. \tag{11}$$

Equation (11) is called the heat balance integral.

Applying Leibnitz' rule on the left-hand side of equation (11) and integrating the right-hand side, one obtains

$$\frac{\mathrm{d}}{\mathrm{d}t}\int_0^{X(t)} T\,\mathrm{d}x - T_f \frac{\partial X(t)}{\partial t} = \alpha \left[\left.\frac{\partial T}{\partial x}\right|_{x=X(t)} - \left.\frac{\partial T}{\partial x}\right|_{x=0} \right]. \tag{12}$$

Combining equation (12) with (10) results in

$$\frac{d}{dt}\int_0^{X(t)} T\,dx = T_\infty \frac{\partial X(t)}{\partial t}$$

$$-(T_f - T_\infty)\frac{h}{\rho c} - \alpha \frac{\partial T}{\partial x}\bigg|_{x=0} \quad (13)$$

Now we assume that the temperature profile within the film can be represented by a second-degree polynomial in the form:

$$T = a + b[X(t) - x] + c[X(t) - x]^2 \quad (14)$$

where the coefficients a, b, c, may depend on time t. Since there are three coefficients in equation (14), three conditions are necessary. Equations (8) and (9) constitute two conditions; the third one is the combination of equations (6), (9), and (10). Substitution and simplification results in the following expression for the temperature profile which is consistent with the boundary conditions

$$T = T_f + \tfrac{1}{2}(T_f - T_\infty)\left[\frac{h}{k}X(t) + F(x) - 2\right]\left[1 - \frac{x}{X(t)}\right] - \tfrac{1}{2}(T_f - T_\infty)\left[\frac{h}{k}X(t) + F(x) - 2 - 2\theta\right]\left[1 - \frac{x}{X(t)}\right]^2. \quad (15)$$

Where

$$F(x) = \sqrt{\left\{\left[\frac{h}{k}X(t) - 2\right]^2 + 8\theta\right\}} \quad (16)$$

and θ is a dimensionless temperature defined as:

$$\theta = \frac{T_w - T_f}{T_f - T_\infty}. \quad (17)$$

Physically θ represents the driving force for the coating process. The coating will not take place at $\theta = 0$ and the larger θ the higher the coating rate.

To find a relationship for the coating thickness, $X(t)$, as a function of time, we integrate equation (13) after substituting the expression for T into it and performing the necessary algebra. The resulting time–thickness relationship is:

$$t = \int_0^{X(t)} \frac{\frac{1}{3\alpha}\left[2\theta + \frac{h}{k}\xi + 5 + \frac{1}{2}F(\xi) + \frac{\frac{h}{k}\xi\left(\frac{h}{k}\xi - 2\right)}{2F(\xi)}\right]}{4\theta + 2 - 3\frac{h}{k}\xi - F(\xi)}\,\xi\,d\xi. \quad (18)$$

The integral in equation (18) can most easily be evaluated by graphical integration.

When the denominator of equation (18) is set to equal zero, the value of the integral, or the time will be infinity. This means that the growth of coating film stops and that the final coating thickness, X_f, is reached. Thus, for final thickness

$$4\theta + 2 - 3\frac{h}{k}X_f - \sqrt{\left[\left(\frac{h}{k}X_f - 2\right)^2 + 8\theta\right]} = 0. \quad (19)$$

Introducing the Biot number, $Bi = (h/k)X_f$, into equation (19) results in

$$4\theta + 2 - 3Bi - \sqrt{[(Bi - 2)^2 + 8\theta]} = 0. \quad (20)$$

Solving equation (20) we get the very simple relationship between the Biot Number and dimensionless temperature

$$Bi = \theta \quad (21)$$

or

$$\frac{h}{k}X_f = \frac{T_w - T_f}{T_f - T_\infty}. \quad (22)$$

From equation (21), we see that, for a given θ, the Biot number is fixed and is equal to it. This means that the final thickness is proportional to k and inversely proportional to h. In other words, a plot of X_f vs. k will give a straight line, while X_f vs. h will yield a hyperbola. Equation (22) may be used for finding the heat-transfer coefficient in the fluidized bed from experimental data. The detailed procedure is described in the Appendix.

Equation (18) contains three dimensionless groups necessary to define the problem. $(h/k)\,X$ $X/\sqrt{(\alpha t)}$ and θ. The group $(h/k)\,X$ is essentially a dimensionless film thickness. However, $X/\sqrt{(\alpha t)}$ is not a convenient group, as it lumps together the dependent variable X with the independent variable t. To overcome this difficulty we eliminate X between the first two groups and come up with a dimensionless time $[h^2/\rho ck]t$. Now we are ready to plot the results of the graphical integration of equation (18) as dimensionless film thickness versus dimensionless time with the dimensionless temperature θ as a parameter. This has been done in Fig. 1. This figure is the most general plot of the solution. Figure 2 is more explicit. It presents

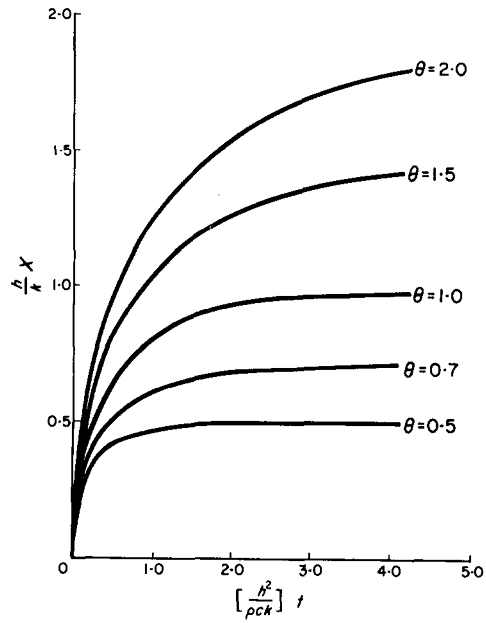

FIG. 1. Dimensionless thickness as a function of dimensionless time with θ as a parameter.

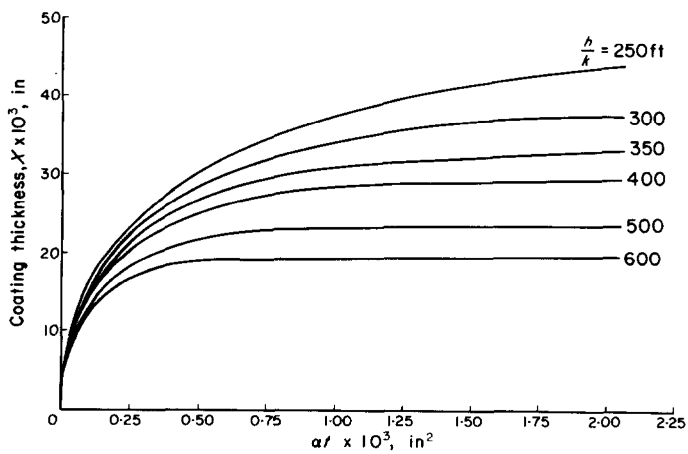

FIG. 2. Plot of coating thickness (X) vs. αt for various h/k and $\theta = 1$.

the coating thickness as a function of αt with h/k as a parameter for the special case of $\theta = 1$. The heat conductivity k is a property of the coating material that cannot easily be changed. The heat-transfer coefficient h, however, can be changed quite readily by changing any one of the parameters of the fluidized bed, such as the air velocity. Figure 2 shows that the effect of the heat-transfer coefficient on the coating thickness is very pronounced. The higher the heat-transfer coefficient, the thinner the coating thickness because of higher heat loss to the surroundings.

To demonstrate the sensitivity of the solution to various parameters, we plotted the coating thickness vs. immersion time for typical working conditions. Figure 3 presents coating thickness

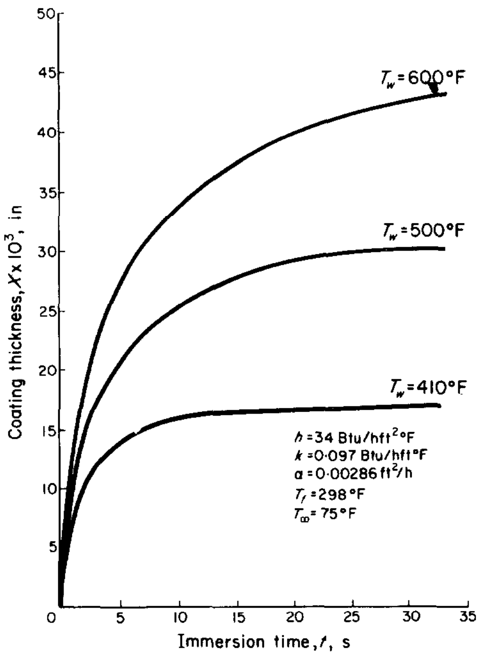

FIG. 3. Effect of immersion time (t) and object temperature (T_w) on coating thickness (X) for a typical coating process.

vs. immersion time curves for various object temperatures. From this Figure it is obvious that the coating thickness is a strong function of object temperature. The object temperature has a more pronounced influence on coating rate in the case of long immersion times.

A plot of final coating thickness, X_f, vs. softening point, T_f, with the heat transfer coefficient as a parameter is shown in Fig. 4. As seen in this figure, the change in final coating thickness with the softening point of the coating material is more pronounced at lower heat-transfer coefficients.

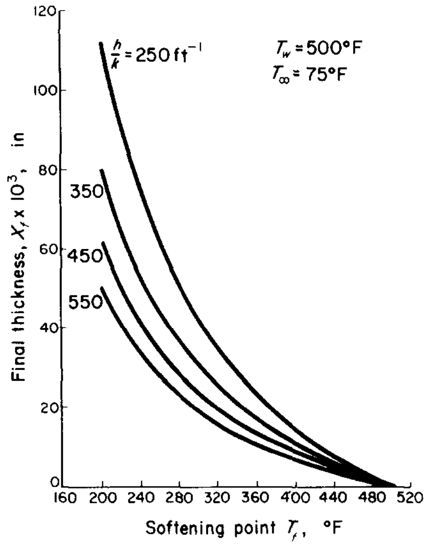

FIG. 4. Final coating thickness (X_f) as a function of softening point of coating material (T_f) for various heat transfer coefficients (h).

From equation (18), it seems that the parameters that affect the coating thickness are only the object temperature, fluidized-bed temperature, and the properties of the coating material. Actually, there are some parameters that affect the coating thickness indirectly because the heat-transfer coefficient is governed by the following factors [5]:

(1) *Properties of the materials*
- (a) Fluidizing gas: thermal conductivity, density, viscosity.
- (b) Fluidized powder: thermal conductivity, shape, size, size distribution, density, specific heat.

(2) *Design of fluidized bed*

Location and geometry of heat transfer surface, size of fluidized bed

(3) *Operating conditions*

Flow rate of fluids, feed or recycle rate of the powder, concentration of the powder in the bed, temperature level and magnitude of the temperature driving forces, etc.

Thus, these variables are indirect factors that can affect the coating thickness.

SOLUTION FOR THE CASE OF NO CONVECTION

In this section, we derive the solution for the coating thickness for the case where heat transfer by convection into the fluidized bed can be neglected. This solution will provide an upper bound on the coating thickness that can be achieved in fluidized-bed coating; that is, one can find what is the maximum coating thickness that can be obtained by changing the condition

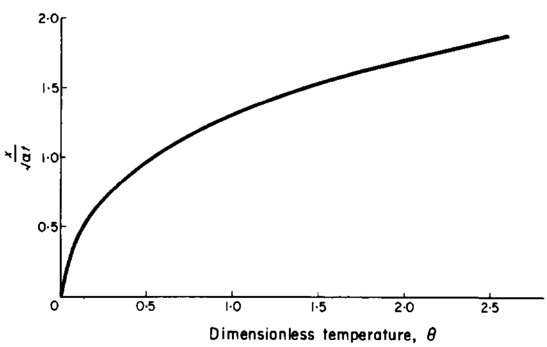

FIG. 5. Dimensionless coating thickness as a function of dimensionless temperature for no heat convection.

of fluidization in the direction of reducing h.

Letting $h = 0$ in equation (18), we get

$$t = \int_0^{x(t)} \frac{(1/3\alpha)\,[2\theta + 5 + \sqrt{(2\theta + 1)}]}{4\theta + 2 - 2\sqrt{(2\theta + 1)}}\xi \mathrm{d}\xi. \tag{23}$$

Integrating and rearranging, we get

$$\frac{x}{\sqrt{(\alpha t)}} = \left[\frac{12\,[2\theta + 1 - \sqrt{(2\theta + 1)}]}{2\theta + 5 + \sqrt{(2\theta + 1)}}\right]^{\frac{1}{2}}. \tag{24}$$

A plot of $X/\sqrt{(\alpha t)}$ vs. θ is shown in Fig. 5. Looking at equation (24), we see that, for a given θ, $X/\sqrt{(\alpha t)}$ is a constant. This means that the

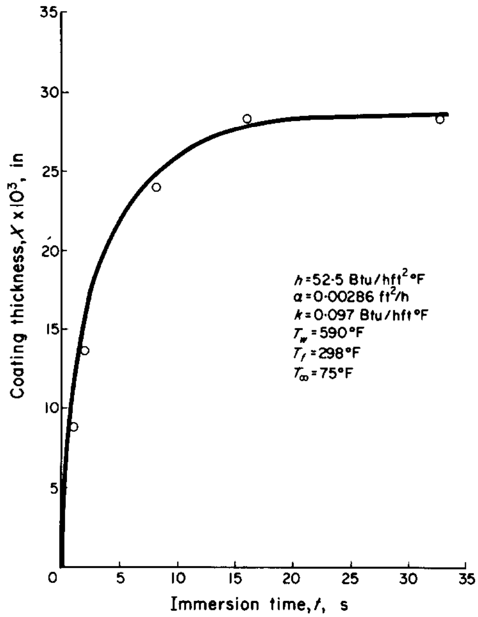

FIG. 6. Plot of coating thickness (X) vs. immersion time (t). Comparison between theory and experimental data of Pettigrew [6].

coating thickness is proportional to the square root of time. For this case there is obviously no final thickness, as for constant object temperature and no heat convection to surroundings the thickness of the film will grow indefinitely.

COMPARISON OF THE THEORETICAL SOLUTION WITH EXPERIMENTAL DATA

The theoretical solution for the coating thickness was developed under the simplified assumptions discussed above. In this section we compare the simplified theoretical solution with some experimental data from the literature. This comparison will show us whether or not the approximate theoretical solution can provide answers for practical coating problems.

Experimental studies of fluidized-bed coating processes were carried out by Pettigrew [6], Richart [7], and Lee [4]. In reporting the experimental data, Pettigrew gave more details on the operating conditions than the others. Thus, the comparison of the theroetical solution and Pettigrew's experimental data is straightforward, while, for the other data, one has to estimate some of the coating parameters.

The experimental data given by Pettigrew are shown as coating thickness vs. immersion time in Figs. 6 and 7 with the coating thickness calculated from theoretical equation (18). The operating conditions are:

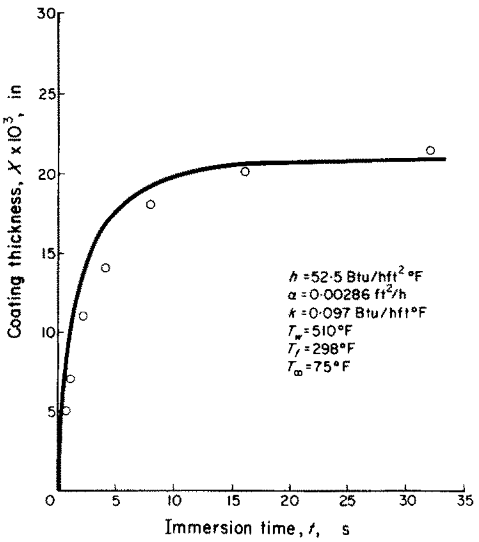

FIG. 7. Coating thickness (X) as a function of immersion time (t). Comparison between theory and experimental data of Pettigrew [6].

1. Coating material.
   The experiments were performed using Corvel vinyl resin VCA-1289.
2. Object material and size.
   The objects were made of $4 \times 3 \times \frac{1}{16}$ in. cold-rolled steel.
3. Preheat cycle.
   The preheat temperature is 650°F for Fig. 6 and 550°F for Fig. 7.
4. Final temperature during coating.
   The object temperature is 590°F for data in Fig. 6 and 510°F for those in Fig. 7.
5. Fluidizing air velocity.
   The air velocity is 4·9 ft/min.

As mentioned above, the heat-transfer coefficient is a major factor in the fluidized-bed coating process. The coating data reported in the literature do not generally give the heat-transfer coefficient in the fluidized bed; thus, we have to estimate the heat-transfer coefficients of fluidized beds from their operating conditions. The heat-transfer coefficient used in the calculation of the theoretical solution from equation (18) as presented in Figs. 6 and 7 is found by a graphical method from experimental data of final coating thickness. The detailed procedures are described in the Appendix. The value of the heat-transfer coefficient used in Figs. 6 and 7 is 52·5 Btu/h ft^2°F. Richart's [7] experi-

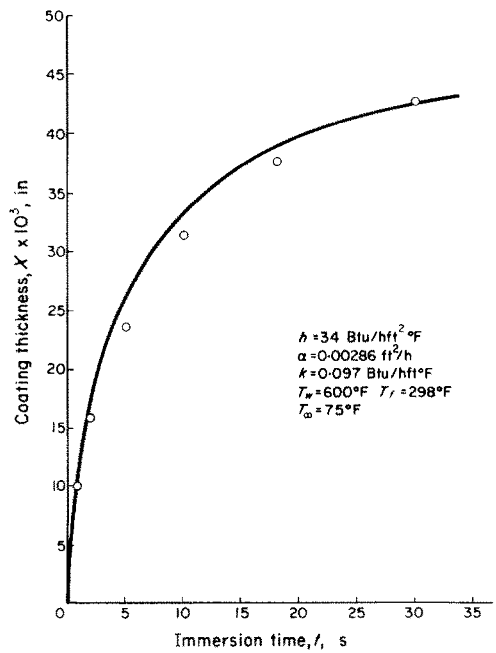

FIG. 8. Plot of coating thickness (X) vs. immersion time (t). Comparison between theory and experimental data of Richart [7].

mental data are shown as plots of coating thickness vs. immersion time in Figs. 8 and 9 together with the curves for coating thickness calculated from equation (18). The operating

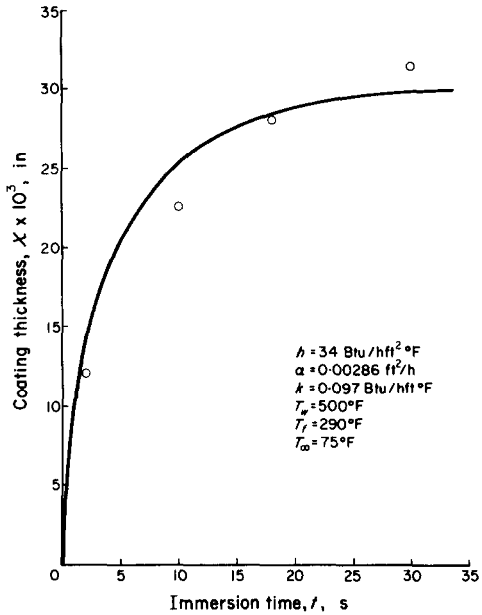

FIG. 9. Effect of immersion time (t) on coating thickness (X). Comparison between theory and experimental data of Richart [7].

conditions are shown in the figures. Since Richart's [7] experimental data do not give final coating thicknesses, we couldn't find the heat-transfer coefficient by a graphical method as we did in the case of Pettigrew's [6] data. Thus, we used the operating conditions given by Richart [7] to estimate the heat-transfer coefficient from the literature. We found the heat-transfer coefficient for similar conditions from Mickley and Trilling [5] as 34 Btu/h ft^2°F and used this value to calculate coating thicknesses shown in Figs. 8 and 9.

As seen in Figs. 6 through 9, the agreement between the theoretical predictions of equation (18) and the experimental data given in the literature is good. For object temperature, T_w = 600°F, the coating thicknesses predicted by theory are, on the average, 10 per cent higher than experimental data; for T_w = 510°F, they are 12 per cent higher. The maximum deviation in coating thickness was less than 30 per cent. The higher thickness predicted by theory is attributed to the assumption of constant object and coating film surface temperatures. Other factors that may account for deviations between the theory and the experiments are the uncertainty of the heat-transfer coefficient, and the temperature profile within the coating film represented by a second-degree polynomial. As seen in Fig. 2, the coating thickness is a strong function of the heat-transfer coefficient. A small change in the heat-transfer coefficient will have a pronounced influence on the coating thickness. Since Richart's [7] operating conditions were not identical to those of Mickley and Trilling [5], the heat-transfer coefficient found from the literature was not very accurate. This could certainly introduce some error into Figs. 8 and 9. One could expect better agreement between theory and experiment if the heat-transfer coefficient data were available. Relaxing the assumption of constant object and coating-film surface temperatures could also improve the validity and range of the theoretical solution, but it would do so at the expense of simplicity.

DISCUSSION AND CONCLUSIONS

In the present study an attempt was made to develop a theoretical relationship between film thickness in fluidized-bed coating and the physical properties of coating material, the object temperature, the fluidized-bed temperature, and the coating time. The theoretical solution was compared with experimental data. As seen in Fig. 6 through 9, it can be stated that this attempt was successful.

The weak points in the developed theory were the following assumptions:

1. The object temperature is constant.
2. The surface temperature of the coating film is constant and equals its softening point.
3. The temperature profile within the coating

film is represented by a second-degree polynomial.

Assumption 1, which probably holds in the case of short immersion times, is questionable for long immersion times.

Assumption 2 is a very questionable assumption in the developed theory. The temperature on the surface of the coating must remain higher than the softening point of the polymer if the coating is to continue to build up.

Following Goodman [2], it is expected that a cubic temperature profile within the coating film would give a considerably more accurate solution than that using Assumption 3.

The following conclusions may be drawn from the present study:

1. The fluidized-bed heat-transfer coefficient is the most significant factor in the coating process. A small change in the heat-transfer coefficient (see Fig. 2) will produce a large change in thickness. The smaller the heat-transfer coefficient, the thicker the coating film. The maximum coating thickness is predicted by equation (25) which was derived for the limiting case of no heat transfer to the fluidized bed.

2. We can control the thickness of the coating film as a function of time by adjusting the object temperature, the fluidized-bed temperature, or the heat-transfer coefficient. The heat-transfer coefficient is governed by the properties of fluidizing gas and fluidized particles, the design of the bed, and the operating conditions. All these design parameters can be controlled and modified to suit the coating problem at hand.

3. All portions of the object to be coated do not have equal residence times in the bath due to immersion and withdrawal time. The easiest way to achieve uniform coatings is to choose working conditions, such that the coating thickness becomes a weak function of time. One, therefore, should work in the flat regions of the curves given in Fig. 1.

4. The predicted coating thickness for higher object temperatures is better than that for lower object temperatures. This is seen in Figs. 6 and 7.

5. The predicted coating thickness is slightly higher than experimental data because of the assumption of constant object and coating surface temperature. Due to the latter assumption, larger deviations are expected with objects having a relatively high surface to volume ratio, such as in thin plates. For these cases, one would have to replace the constant object temperature with a total heat balance performed on the object. Although the problem appears easy in principle, it is involved enough as to prevent us from obtaining a simple solution to it. Thus, this problem is left for a future effort.

We feel that at present the solution obtained is accurate enough and simple enough in order to attract the practising coating technologist. On the other hand, it is basic in its approach and the assumptions made, though robust, are reasonable and pass the test of experiment.

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APPENDIX

Method of Finding Heat-Transfer Coefficients from Experimental Data

As mentioned above, if data of the final thickness of film vs. object temperature are available, we can find the heat-transfer coefficient, h, directly from the experimental data. The discussion presented in this Appendix provides a method of finding the heat-transfer coefficient from experimental data.

Under the assumption we made above, the energy balance equation at the surface of the coating film is

$$-k\frac{\partial T}{\partial x} = h(T_f - T_\infty) + \rho c(T_f - T_\infty)\frac{\partial X(t)}{\partial t}. \quad \text{(A.1)}$$

When the coating film reaches its final thickness, the growth of coating film stops and equation (A.1) becomes

$$-k\frac{\partial T}{\partial x} = h(T_f - T_\infty). \quad \text{(A.2)}$$

Equation (A.2) expresses the fact that heat conduction at the surface of the coating film equals the heat flow into the fluidized bed by convection.

Integrating equation (A.2) from $x = 0$ to $x = X_f$, and rearranging, we obtain

$$T_w = T_f + (T_f - T_\infty)\frac{h}{k}X_f. \quad \text{(A.3)}$$

Equation (A.3) can also be obtained by rearranging equation (22).

If we plot the object temperature T_w, versus the final thickness X_f, a straight line is obtained. The intercept of this line yields T_f and its slope—the heat-transfer coefficient h.

Résumé—Cette étude présente une relation théorique entre l'épaisseur du revêtement sur un objet immergé dans un lit fluidisé du matériau du revêtement et les propriétés physiques du système.

La théorie du revêtement par lit fluidisé a été exposée en employant l'intégrale du bilan de chaleur pour résoudre le problème de la conduction de la chaleur avec une frontière mobile appliqué au revêtement. Dans l'exposé de la théorie, le profil de température dans le revêtement a été représenté par un polynôme du second degré.

La solution théorique a été comparée avec un bon accord aux résultats expérimentaux donnés dans la littérature. En moyenne, la théorie prédit des épaisseurs de revêtement qui sont plus élevées que les valeurs expérimentales d'environ 10 pour cent. Cette déviation est attribuée principalement à l'hypothèse faite dans la théorie d'une température constante de l'objet et de la surface du revêtement.

Le coefficient de transport de chaleur est un facteur principal dans le processus de revêtement par lit fluidisé. Les résultats sur le revêtement, signalés dans la littérature ne donnent pas généralement le coefficient de transport de chaleur dans le lit fluidisé. Une méthode graphique pour estimer le coefficient de transport de chaleur a été exposée pour des expériences où l'on donne les résultats pour l'épaisseur finale.

Zusammenfassung—Diese Untersuchung liefert die theoretische Beziehung zwischen der Beschichtungsdicke eines Körpers, der in ein Fliessbett aus Beschichtungsmaterial getauscht ist und den physikalischen Eigenschaften des Systems. Die Theorie der Fliessbettbeschichtung ergab sich mit Hilfe des Wärmebilanzintegrals bei der Lösung des Problems der Wärmeleitung mit bewegter Grenze für die Beschichtung. In der Theorie wurde das Temperaturfeld in der Beschichtung durch ein Polynom zweiten Grades wiedergegeben.

Die theoretische Lösung wurde mit experimentellen Werten aus der Literatur verglichen. Die Übereinstimmung ist gut. Im Mittel liefert die Theorie Schichtdicken, die um etwa 10 Prozent grösser sind als die experimentellen. Diese Abweichung wird vorwiegend der Annahme konstanter Körper- und Schichtoberflächentemperatur zugeschrieben.

Der Wärmeübergangskoeffizient ist ein Hauptfaktor im Fliessbettbeschichtungsprozess. Die in der Literatur gegebenen Beschichtungsdaten liefern gewöhnlich nicht den Wärmeübergangskoeffizienten im Fliessbett. Eine grafische Methode zur Abschätzung des Wärmeübergangskoeffizienten wurde für Versuche mit angegebenen Werten der Enddicke entwickelt.

Аннотация—В данном исследовании получено теоретическое соотношение между толщиной покрытия предмета, погруженного в псевдоожиженный слой материала покрытия, и физическими свойствами системы.

Теория покрытия в псевдоожиженном слое разработана на основе интегрального теплового баланса при решении задачи теплопроводности с подвижной границей применительно к пленке покрытия. При этом распределение температуры внутри пленки покрытия представлено полиномом второй степени.

Сравнение теоретического решения с экспериментальными данными, приведенными в литературе, показало хорошее соответствие. Расчетная толщина покрытия превышает экспериментальное значение в среднем на 10%. Это отклонение объясняется, в основном, принятым в теории допущением о постоянстве тепмературы поверхности изделия и пленочного покрытия.

Коэффициент теплообмена является основным фактором в процессе покрытия в псевдоожиженном слое. В литературных данных по покрытиям коэффициент теплообмена в псевдоожиженном слое обычно не приводится. Разработан графический метод расчета коэффициента теплообмена для экспериментов, в которых приводится окончательная толщина покрытия.